S1 File. Model equations.

An R package for simulating growth and organic wastage in aquaculture farms in response to environmental conditions and husbandry practices

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Table A. Model state variables, forcings, and functional relationships of *Mytilus galloprovincialis* – as in Brigolin et al. (2009)

Prognostic state variables W_b : somatic dry weight [g] R: gonadic dry weight [g] **Diagnostic state variables** W_d : dry weight of the mussel [g] W_f: wet weight of the mussel [g] W_{tot} : total weight of the mussel, including the shell [g] L: length of the shell [cm]	**1. Growth equations** $\frac{dW_b}{dt} = (1-k) \cdot \frac{(A\text{-}C)}{\varepsilon_B}$ (1) $\frac{dR}{dt} = k \cdot \frac{(A\text{-}C)}{\varepsilon_R}$ (2) **2. Computation of the available energy** $Cp = chl2cp \cdot CHL$ (3) $PHY = \frac{Cp}{\gamma}$ (4) $DT = POC - Cp$ (5)
Forcings T_w : water temperature [°C] POC: Particulate Organic Carbon concentration [mgC l^{-1}] CHL: Chlorophyll-a concentration [mg l^{-1}] PHY: Phytoplankton concentration [mg l^{-1}] Cp: Phytoplankton-C concentration [mgC l^{-1}] TSM: Total Suspended Matter concentration [mg l^{-1}] POM: Particulate Organic Matter concentration [mg l^{-1}] DT: Organic detritus concentration [mg l^{-1}] **Parameters** $chl2cp$: Conversion factor from CHL to Cp [-] γ: Conversion factor from Cp to Phy [-] AE_{max} : Maximum Adsorption Efficiency [-] K_s : Half-saturation constant for the AE [-] T_{ma} : Maximum temperature for the anabolic processes [°C] T_{oa} : Optimal temperature for the anabolic processes [°C] β_a : Temperature exponent for the anabolism [1/°C] Cr_{max} : Maximum Filtration rate [l/(day·gDW)]	**3. Functional expressions for net anabolism** $f_a = \left(\frac{T_{ma} - T_w}{T_{ma} - T_{oa}}\right)^{\beta_a \cdot (T_{ma} - T_{oa})} \cdot e^{\beta_a \cdot (T_w - T_{oa})}$ (6) $I = CR_{\max} \cdot f_a \cdot W_b^q \cdot \left(\varepsilon_{DT} \cdot DT + \varepsilon_{Phy} \cdot PHY\right)$ (7) $Q = \frac{POM}{TSM}$ (8) $AE = AE_{\max} \cdot \frac{Q}{Q + K_s}$ (9) $E = AE \cdot I$ (10) $A = (1-\alpha) \cdot E$ (11) **4. Functional expressions for fasting catabolism** $f_c = \left(\frac{T_{mc} - T_w}{T_{mc} - T_{oc}}\right)^{\beta_c \cdot (T_{mc} - T_{oc})} \cdot e^{\beta_c \cdot (T_w - T_{oc})}$ (12) $C = R_{\max} \cdot f_c \cdot \varepsilon_{O_2} \cdot W_b$ (13) **5. Reproduction events** $R(t = spawn_1) = R(t = spawn_2) = 0$ (14) **6. System output** $W_d = W_b + R$ (15) $W_f = a_f \cdot W_d$ (16) $W_{tot} = a_{tot} \cdot W_d$ (17)

q: Weight exponent for filtration [-] ε_{DT} : Energy content of detritus [J/mg] ε_{PHY} : Energy content of phytoplankton [J/mg] α: Feeding catabolism [-] A_{max} : Maximum energy ingestion rate for a 1 g mussel [J/(g·day)] T_{mc} : Maximum temperature for the catabolic processes [°C] T_{oc} : Optimal temperature for the catabolic processes [°C] β_c : Temperature exponent for the catabolism [1/°C] R_{max} : Maximum respiration rate [mgO2/(gDW·day)] ε_{O2} : Energy consumed by the respiration of 1g of oxygen [J/mgO2] k: Energy fraction invested in reproduction [-] ε_B : Somatic tissue energy content [J/g] ε_R : Gonadic tissue energy content [J/g] a_f: Dry weight-wet weight conversion coefficient [-] a_{tot} : Dry weight-total (including shell) weight conversion coefficient [-] a_L : Weight-length conversion coefficient [mm/mg^{bL}] b_L : Shape-coefficient for the weight-length conversion [-]	$L = a_L \cdot W_d^{\,b_L}$ (18)

Table B. Functional expressions used in the individual growth models of *Sparus aurata* and *Dicentrarchus labrax* – as in Brigolin et al. (2010; 2014)

State variable:

w: fresh weight [g]

Growth equation:

$$\frac{dw}{dt} = \left(\frac{A - C}{\varepsilon_T} \right);$$

A: net anabolism [J day^{-1}]

C: fasting catabolism [J day^{-1}]

ε_T: energy content of somatic tissue [kJ g^{-1}]

Forcings:

T_w : water temperature [°C]

R: amount of food provided by the farmer per individual [g day^{-1}]

C_P : % of proteins in the ingested food

C_C : % of carbohydrates in the ingested food

C_L : % of lipids in the ingested food

Functional expressions for net anabolism

$$I = I_{max} \cdot H(Tw) \cdot w^m$$

I: daily ingestion rate [g day^{-1}]

I_{max}: maximum ingestion rate [g day^{-1} g^{-m}]

m: weight exponent for the anabolism

F: faeces production [g day^{-1}]

$$\begin{cases} I = R & , when\ I \geq R \\ I = 0 & , when\ T < T_a \end{cases}$$

$$A = (1-\alpha) \cdot I \cdot [C_P \cdot \varepsilon_P \cdot \beta_P + C_C \cdot \varepsilon_C \cdot \beta_C + C_L \cdot \varepsilon_L \cdot \beta_L]$$

$$F = I \cdot [C_P \cdot (1-\beta_P) + C_C \cdot (1-\beta_C) + C_L \cdot (1-\beta_L)]$$

α: feeding catabolism coefficient

β_P, β_C, β_L: assimilation coefficient for protein, carbohydrate and lipid

ε_P, ε_C, ε_L: energy content of protein, carbohydrate and lipid [kJ g^{-1}]

2. Functional expressions for fasting catabolism

$$C = \varepsilon_{O2} \cdot k_0 \cdot K(T_w) \cdot w^n$$

ε_{O2}: energy consumed by the respiration of 1 g of oxygen [kJ g^{-1}]

k_0: fasting catabolism at 0°C [day^{-1} g^{-n}]

n: weight exponent for the catabolism

$$H(T_w) = \left(\frac{T_m \text{-} T_w}{T_m \text{-} T_o}\right)^{b(T_m \text{-} T_o)} \cdot e^{b(T_w \text{-} T_o)}$$

b: shape coefficient for the $H(T_w)$ function

T_o: optimal temperature [°C]

T_m: maximum lethal temperature [°C]

$$K(T_w) = e^{pk \cdot T_w}$$

pk: temperature coefficient for the fasting catabolism [$°C^{-1}$]

O: daily respiration rate [day^{-1}]

$Ex_{P,N}$: daily dissolved N,P excretion rates [day^{-1}]

$$O = k_0 \cdot K(T_w) \cdot w^n$$

$$Ex_N = O \cdot k_{N:O}$$

$$Ex_P = O \cdot k_{P:O}$$

3. Wasted feed (W) W: uneaten feed [g day^{-1}] $\begin{cases} W = R - I & ,when\ R \geq I \\ W = 0 & ,when\ R < I \end{cases}$

Table C. Model state variables, forcings, and functional relationships of *Ruditapes philippinarum* - as in Solidoro et al. (2000)

W_w : wet weight [g] W_d : dry weight [g] L: length of the shell [mm] b: coeff. of allometric equation relating w_d to w_w a: coeff. of allometric equation relating w_w to L	**Isometric relation** $W = aL^3$ **Allometric relation** $w_d = bw_w^p$
T: water temperature [°C] F: Food concentration in water [µg chl-a l^{-3}] G_{wmax} : Max. growth rate on a wet weight basis [gww$^{1/3}$ day^{-1}] r_{wmax} : Max. respiration rate on a wet weight basis [day^{-1}]	**Growth equations** if E > E*, which is equivalent to F > F* $\frac{dL}{dt} = G_{L\max} f_{gT}(T) f_{gF}(F) - r_{L\max} f_{rT}(T) L$ $\frac{dw_w}{dt} = G_{w\max} f_{gT}(T) w_w^{2/3} - r_{w\max} f_{rT}(T) w_w$

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G_{dmax} : Max. growth rate on a dry weight basis [gdw$^{0.265}$ day^{-1}]
r_{dmax} : Max. respiration rate on a length basis [day^{-1}]
G_{Lmax} : Max. growth rate on a length basis [mm day^{-1}]
r_{Lmax} : Max. respiration rate on a length basis [day^{-1}]
p: Coeff. of allometric growth equation relating w_d to w_w [-]
q: Coeff. of allometric filter velocity [-]
T_{mG} : Max. temperature for growth [°C]
T_{oG} : Optimal temperature for growth [°C]
T_{mr} : Max. temperature for respiration [°C]
T_{ov} : Optimal temperature for filtration [°C]
ε_F : Energetic content food [J µg chl-a^{-1}]
ε_T : Energetic content of *Ruditapes philippinarum* [J g dw^{-1}]
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$\frac{dw_d}{dt} = G_{d\max} f_{gT}(T) w_d^{(1-1/3p)} - r_{d\max} f_{rT}(T) w_d$
if E ≤ E*, which is equivalent to F ≤ F*
$\frac{dL}{dt} = \frac{F}{F^*} G_{L\max} f_{gT}(T) f_{gF}(F) - r_{L\max} f_{rT}(T) L$
$\frac{dw_w}{dt} = \frac{F}{F^*} G_{w\max} f_{gT}(T) w_w^{2/3} - r_{w\max} f_{rT}(T) w_w$
$\frac{dw_d}{dt} = G_{d\max} f_{gT}(T) w_d^{(1-1/3p)} - r_{d\max} f_{rT}(T) w_d$
$= FV_f f_v(T) w_d^q \frac{\varepsilon_F}{\varepsilon_T} - r_{d\max} f_{rT}(T) w_d$
with
$G_{d\max} = pG_{w\max} b^{(1/3p)} = pG_{r\max} 3a^{1/3} b^{(1/3p)}$
$r_{d\max} = pr_{w\max} = 3rp_{r\max}$
Threshold below which growth is food limited
$F^* = \frac{G_{d\max} f_{gT}(T) w_d^{(1-1/3p)} \varepsilon_T}{V_f f_v(T) w_d^q \varepsilon_F}$
Functional response of temperature for growth, respiration and filtration
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$f_{gT}(T) = (\frac{T_{mG} - T}{T_{mG} - T_{oG}})^{\beta_G (T_{mG} - T_{oG})} e^{\beta_G (T - T_{oG})}$
$f_{tT}(T) = (\frac{T_{mr} - T}{T_{mr} - T_{or}})^{\beta_r (T_{mr} - T_{or})} e^{\beta_r (T - T_{or})}$
$f_v(T) = (\frac{T_{mv} - T}{T_{mv} - T_{ov}})^{\beta_v (T_{mv} - T_{ov})} e^{\beta_v (T - T_{ov})}$
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